

An Algorithm for Evaluating Changes in Linear Programming using Kuhn Tucker condition¹EGBON, F.O., ²OLOFINLAE, O. O.^{1,2}Department of Mathematics, College of Education, Ikere-Ekiti, Nigeria¹E-Mail Id: egbon.friday@yahoo.com²E-Mail Id: latjaylove1591@yahoo.com**Abstract**

This paper examined an Algorithm for evaluating changes in Linear Programming using Kuhn Tucker condition. When changes are made in linear programming model, we sought for an optimal solution using an algorithm to determine the final value of the problem involving Kuhn Tucker (K-T) conditions for a basic feasible solution. There exists, a flowchart of the algorithm as presented in this paper to enhance the understanding of the application.

Keywords: Algorithm, changes, linear, programming, linear programming

Introduction

Many parameters in system dynamics models represent qualities that are very difficult or even impossible to measure to a great deal of accuracy in reality. Some parameter values change in the real world and these analyses allow one to determine what level of accuracy is necessary for a parameter to make the model sufficiently useful and valid.

For every maximization linear programming problem, there is a corresponding maximization of profit which is equivalent to the minimization of the cost of operating the business. These pairs of closely related problems are called dual linear programming problems. The first problem for which dual is sought is referred to as the primal problem, which is the LP model developed for a particular situation. The other related one is the dual one which is a closely related Mathematical problem that can be derived directly from the primal problem. The rules for formulating the symmetric or nonsystematic dual LP problem from the primal LP problem and otherwise are contained in Ekoko (1999). Thus, in an LP model in which the number of variables is considerably smaller than the number of constraints, computational savings may be realized by solving the dual from which the primal solution is automatically determined. As a result, the objective value may continue to increase (maximization case) indefinitely. In this case, both the solution space and optimum objective value are unbounded (Taha, 2006). The practical application of linear programming can be hampered at times by imperfect knowledge of the necessary data or by a complete lack of data. Ekoko (2004) proposed an algorithm for determining the final maximum of the θ considered to be a linear function of a parameter.

The Linear Programming Problem

$$\text{Maximize } Z = \sum_{j=1}^n C_j X_j$$

Subject to

$$\sum_{j=1}^n a_{ij} X_j \leq b_i, i = 1, 2, \dots, m \text{ and } X_j \geq 0, j = 1, 2, \dots, n,$$

Where C_j 's are the cost coefficients, b_i is right-hand side (R.H.S) values which are the technological constraints coefficients. Changes are most commonly investigated in the objective functions and in the right-hand side values b_1, b_2, \dots, b_m . These may correspond to changes in the availability of raw materials or changes in the quality and quantity of goods produced. The detection of these outcomes is by critical analysis which is mainly on the primal-dual properties (Forester, 2001). Researchers who have been in search of computational algorithms for determining the maximum value of the parameter earlier stated, are in critical need of the one presented in this work.

When graph is used to solve linear programming problem, the use of slack and surplus variables do not arise. However, there is a theory called Kuhn-Tucker (K-T) conditions which states that any primal constraint for which equality does not hold at optimal solution, will have the value of its corresponding dual variable zero (Agbadudu, 1996). Thus, any constraint that is **NOT BINDING** will have dual variable equal to zero. But a constraint in the primal problem for which equality holds at optimal solution will have its corresponding dual variable different from zero. Note that this method is used in the absence of simplex tables. An algorithm that could observe continuous changes in the cost coefficient of a linear programming problem is proposed using Kuhn-Tucker (K-T) conditions. The algorithm is intensive and a formal presentation of the steps with some explanations is given below:

Step 1: Involves expressing the primal problem

$$\text{Maximize } Z = \sum_{j=1}^n C_j X_j$$

s.t

$$\sum_{j=1}^n a_{ij} X_j \leq b_i, i = 1, 2, \dots, m, X_j \geq 0, j = 1, 2, \dots, n$$

in its computational form (i.e. by putting the constraint in equation form without slack or surplus variable and initializing the optimum of the primal and the optimum of the dual problem).

Step 2: Compute for the variables using the

$$\sum_{j=1}^n a_{ij} X_j \leq b_i, i = 1, 2, \dots, m \text{ and } X_j \geq 0, j = 1, 2, \dots, n$$

Step 3: Determine the optimum solution of the original primal problem using the equation of the objective function.

$$\text{Maximize } Z = \sum_{j=1}^n C_j X_j, j = 1, 2, \dots, n$$

Step 4: Verify if the optimum solution is equal to the optimum dual solution. If yes, Go to step 9 else continue

Step 5: Convert to the corresponding dual problem.

$$\text{Maximizing } Y_o = \sum_{j=1}^n b_j y_j$$

$$\sum_{j=1}^m a_{ij} Y_j \geq C_j, j = 1, 2, \dots, m$$

$$y_j \geq 0, j = 1, 2, \dots, m$$

Step 6: If the constraint is binding therefore, the variable is different from zero (0) else equate the variable to zero (0) Go to step 9.

Step 7: Compute the value of the variable of the dual problem

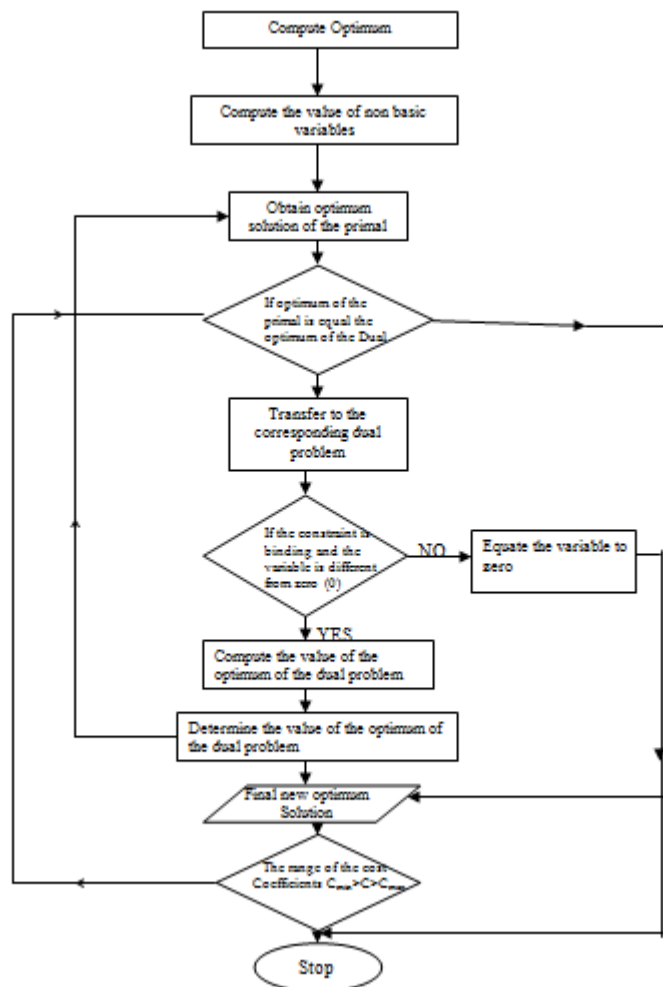
Step 8: Determine the value of the optimum of the dual solution and Go to step 4

$$y_o = \sum_{j=1}^m b_j y_j, \quad j = 1, 2, \dots, m$$

Step 9: Display final solution

Step 10: If there are changes in the cost coefficients i.e the cost coefficients are open and termination point, take C_{max} as final, else Go to step 3

Flowchart of the algorithm



A proof to help us have a better understanding of the steps in the algorithm

$$X_o = \sum_{j=1}^n C_j X_j$$

$$X_o \leq \sum_{j=1}^n C_j X_j \left[\sum_{i=1}^m a_{ij}, Y_i \right] x_j$$

$$X_o \leq \sum_{j=1}^n \left[\sum_{i=1}^m a_{ij}, X_j \right] y_j$$

$$X_o \leq \sum_{i=1}^m b_i Y_i = y_o$$

Hence, the primal problem

Maximize $X_o = \sum_{j=1}^n C_j X_j$

Subject to

$$X_o = \sum_{j=1}^n a_{ij} X_j \leq b_i \quad i = 1, 2, \dots, m$$

$$X_j \geq 0, \quad j = 1, 2, \dots, n$$

Can be transformed to its associated dual in the form

Maximize $Y_o = \sum_{j=1}^m b_j Y_j$

Subject to

$$\sum_{j=1}^n a_{ij} Y_i \geq C_j \quad J = 1, 2, \dots, n$$

$$Y_i \geq 0, \quad i = 1, 2, \dots, m$$

Numerical Illustration

Primal problem:

Minimize $Z = 100X_1 + 125X_2$

Subject to

$$5X_1 + 10X_2 \geq 400$$

$$3X_1 + X_2 \geq 100$$

$$2X_1 + 15X_2 \geq 270$$

$$X_1, X_2 \geq 0$$

Dual problem

Maximize $W = 400Y_1 + 100Y_2 + 270Y_3$

Subject to

$$5Y_1 + 3Y_2 + 2Y_3 \geq 100$$

$$10Y_1 + Y_2 + 15Y_3 \geq 125$$

$$Y_1, Y_2, Y_3 \geq 0$$

When the primal problem is solved using simplex or graphical method, we obtain the values $X_1 = 24, X_2 = 28$ and $Z = 5900$. Now, substituting the values of X_1 and X_2 in the first constraint of the primal problem, we have $5(24) + 10(28) = 120 + 280 = 400$.

The constraint is binding and therefore, Y_1 is different from zero. The second constraint is also binding, hence Y_2 is different from zero. But the third constraint is $2(24) + 15(28) = 48 + 360 = 408$ which is different from 270, the right hand side of that constraint. The third constraint is not binding and hence Y_3 is zero. With Y_3 being zero(0), the dual constraints now reduce to

$$5Y_1 + 3Y_2 = 100$$

$$10Y_1 + Y_2 = 125$$

Solving, we have $Y_1 = 11$ and $Y_2 = 15$. The solution to the dual problem is $Y_1 = 11, Y_2 = 15$ and $Y_3 = 0$. The value of the objective function $W = 400Y_1 + 100Y_2 + 270Y_3$ becomes $W = 400(11) + 100(15) + 270(0) = 4400 + 1500 + 0$ which is 5900 which is equal to Z in the primal problem.

Conclusion

In several problems of life confronting man, we want to find out not only an optimal solution but also the fluctuating cost of raw materials, expected sales volumes, labour cost etc. What we want to know from the analysis is which constraint is binding or not binding in step 6 with respect to any primal constraint for which equality ceases to hold at optimal solution, will have the value of the corresponding dual variable zero (0), but any constraint in the primal problem for which equality holds at optimal solution will have its corresponding dual variable different from zero.

Then we can concentrate on getting accurate data for those items or at least running several with various values of the crucial data in place of generate an idea of the range of possible outcomes. Since we live in a dynamic world where constraints are inevitable, the study of the algorithm and flowchart on the solution due to changes in the data of a problem is very useful to observe continuous changes in the cost coefficient of a linear programming problem using Kuhn & Tucker (K-T) conditions.

Recommendation

This study has enable us to understand the dynamic behavior of a linear programme with respect to range of values in which certain decision variables can take without violating optimality and feasibility conditions. Hence, this algorithm is recommended to confirm rigorous and iterated computation in a complex real life situation.

References

- [1]. Agbadudu, A.B. (1996). *Elementary Operations research*. Vol.1, Benin City: Mudiaga Ltd.
- [2]. Ekoko, P.O. (1999). *Basic Operation Research in science and social sciences*. Benin City, United City press.

- [3]. Ekoko, P.O. (2004). "An algorithm for evaluating continuous changes in Right hand side (R.H.S) values of LP problem", *Journal for die reine Und Angewandte Mathematic*, 6, 1189-1194.
- [4]. Forrester, J.W. (2001). *An introduction to sensitivity Analysis*. MIT System Dynamics in Education project.
- [5]. Kuhn, H. & Tucker, A. (1951) "*Nonlinear Programming*". In Berkeley Symposium on Mathematical Statistics and Probability, University of California Press, Berkeley and Los Angeles.
- [6]. Taha, R.A. (2006). *An introduction to operational research*. New Delhi prentice Hall of India.